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The impact of loan prepayment risk and deposit withdrawal risk on the optimal intermediation margin

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Abstract

Numerous studies have analyzed how a bank's intermediation margin varies with respect to such factors as credit quality, funding risk, bank capital, deposit insurance and other factors. However, these studies ignore the potential that loans tend to prepay if interest rates decline and deposits tend to be withdrawn if interest rates rise. Taking this very fundamental fact into account, we derive optimal loan rates and deposit rates when the bank is subject to loan prepayments and deposit withdrawals. Among other things, we find that greater volatility of interest rates tends to increase the margin. The strength of the correlation between the level of interest rates and the propensity to prepay loans (withdraw deposits) also plays an interesting role.

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1. Introduction

At their most fundamental level, banks are in the business of borrowing and lending money. In this context, one popular way of characterizing a bank is as a market

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maker or dealer. In Ho and Saunders (1981), banks are dealers in loan and deposit markets where the major difficulty in managing the process is that loans and deposits arrive stochastically and not at the same time. [1] The authors show that a bank will charge an intermediation fee for the immediate provision of loan and deposit accounts to its customers. This intermediation margin is shown to be dependent upon management's degree of risk aversion, the bank's market structure, the average size of bank transactions, and the variance of interest rates. Subsequent papers have also focused on the optimal intermediation margin as it relates to various types of uncertainty which are common to the banking environment. Wong (1997) analyzes the intermediation margin under interest rate risk and finds it positively related to market power, operating costs, and risk aversion. [2] Wong contrasts his model with others such as Zarruk (1989) where the sole source of uncertainty is funding risk and Madura and Zarruk (1992) where credit is risky. Zarruk (1989) finds that increases in bank capital typically increase the intermediation margin while deposit volatility reduces the margin. Madura and Zarruk (1992) find that increases in bank capital requirements and deposit insurance premiums reduce borrowing and lending margins. Allen (1988) has analyzed the impact of cross elasticities of bank products upon the intermediation margin; the margin is shown to be dependent on monopoly power, a risk premium and multi-product diversification. Angbazo (1997) has empirically confirmed that banks with riskier loans and higher interest rate risk exposure enjoy larger intermediation margins.

The studies given above have undoubtedly enhanced our understanding of bank intermediation margins but an important aspect of loan and deposit pricing has been ignored in the literature. Specifically, this is the ability of bank customers to exercise their commonly held options to prepay loans and to withdraw deposits. The purpose of this research is to detail the impact of the embedded options upon the optimal intermediation margin of banks by analyzing optimal loan and deposit rates. In our analysis, we formulate the objective function such that the revenues from making a loan will depend upon a rational process for prepayments by borrowers. Similarly, the bank's cost of acquiring funds will depend on a rational process for withdrawals by depositors. The propensity of borrowers to prepay their loans is characterized as

[1] The literature on loan rate determination is extensive and many authors have considered deposit rate setting behavior of banks. Indeed, Baltensperger (1980) and Santomero (1984) provide reviews of these two segments of research on commercial banking theory. However, the determination of the spread between the rate that banks receive on loans and pay for deposits has drawn relatively less attention in the literature and is considered in our introduction.

[2] Interest rate risk can be defined and measured in various ways according to the model being developed. In our case, interest rate risk refers to changes in interest rates that lead to loan prepayments or deposit withdrawals. Wong's (1997) measure is represented by fixed rate loans being financed by variable rates deposits. Similarly, McShane and Sharpe (1985) represent interest rate risk by the amount loan and deposit volume differ; the difference is covered by borrowing or lending in the short term market. Ho and Saunders (1981) measure interest rate risk as the volatility of interest rates that can shrink the interest margin in a utility maximization model. In Angbazo (1997), interest rate risk is measured by net short term assets divided by bank equity.

being inversely related to the level of interest rates, while depositor propensity to withdraw funds is positively related to the level of interest rates. The notional propensity to prepay, however, is only realized if interest rates fall below the lending rates charged to borrowers, R_{L}. Funds are withdrawn early only if the level of interest rates rise above the rate paid to depositors, R_{D}. Clearly, the rates banks charge borrowers and pay depositors influence not only their profitability directly through an instantaneous spread but also via the likelihood bank customers will exercise their implicit options. Any change in R_{L} changes the bank's total revenue and its risk of prepayment while any change in R_{D} changes the bank's total cost and its risk of early withdrawal. [3]

It is important to stress that the balance sheet structure of depository institutions such as banks is a critical motivation for this research. In essence, banks are short options on both sides of the balance sheet such that their situation is similar to that of an option strategy termed a short straddle. That is, after making a loan and securing deposits, the bank would prefer that interest rates remain stable because if rates fall, loans will be called and replaced with lower yielding bank assets. Similarly, if rates rise, deposits will be replaced with higher yielding liabilities. Alternatively stated, loan and deposit rates are characterized by mismatched propensities to prepay and withdraw where exercise of either prepayment or withdrawal are to the disadvantage of the bank.

We derive the optimal output price, R_{L}^*, and the optimal input price, R_{D}^*, assuming the bank is a monopolist in the loan market and a monopsonist in the market for deposits. [4] Then we detail the impact upon R_{L}^* of infinitesimal increases in the following parameters: the mean propensity to prepay, the mean level of interest rates, the standard deviation of the propensity to prepay, the standard deviation of interest rates and the correlation coefficient between these two variates. We also consider the comparative static behavior of R_{D}^* for an alternative but overlapping set of parameters. Finally, we summarize the impact of all the parameters upon the intermediation margin. In some cases the margin will clearly increase or decrease with respect to a change in a particular parameter but in

[3] Ahn et al. (1999) minimize the firm's risk by determining the optimal exercise price on options the firm issues. Similarly, this paper could be considered a risk management paper where the bank seeks to manage the risks of loan prepayment and early deposit withdrawal. Optimal loan and deposit rates are the optimal striking prices set by the bank as the issuer of the implicit options on loans and deposits.

[4] The bank is faced with a downward sloping demand schedule so more loans are placed with borrowers only if the rate of interest charged, R_{L}, is reduced. The supply of deposits schedule is positively sloped so that the bank can secure more deposit funds only by increasing R_{D}. The bank is a monopolist in regard to selling its output and a monopsonist when acquiring more inputs. Our assumption that banking markets are not perfectly competitive is consistent with a large number of published articles which are too numerous to list here. Four examples among many are given below. Hannan (1991) finds local commercial loan markets vary in their competitiveness which helps explain differences in loan rates. Berger and Hannan (1998) find cost inefficiencies in less competitive, more concentrated banking markets. Berger et al. (2000) find persistence in bank profits is partially explained by bank market power in output markets. Hutchison and Pennacchi (1996) examine pricing of retail deposits in imperfect markets.

others the sign of the impact is dependent on the supply, demand and risk elasticities at hand.

2. Objective function and the optimal lending rate

If x is the level of interest rates with upper and lower bounds of K and k, y is the propensity to prepay loans, R_{L} is the interest rate charged on loans, and L is the volume of loans made at R_{L}, then the impact of loan prepayments upon the total revenue function is given quite simply as

$$\mathrm{TR} = LR_{\mathrm{L}} - h(\cdot), \tag{1}$$

where

$$\begin{aligned} h &= 0 && \text{if } R_{\mathrm{L}} \leqslant x \leqslant L, \\ h &= (R_{\mathrm{L}} - x)yL && \text{if } k \leqslant x < R_{\mathrm{L}}. \end{aligned}$$

If the realization of x rises above R_{L} then no prepayments will take place and total revenue will be undisturbed by the interest rate change. Alternatively, if the realization for the random variable x falls below R_{L}, then prepayments take place in a proportion y and they impact total revenue. The bank's forthcoming revenue falls by $R_{\mathrm{L}}Ly$ but the bank will mitigate the loss by reinvesting the prepaid principal at x to earn xLy. The total revenue, in this case, would be $LR_{\mathrm{L}}(1 - y) + xyL$.

If w is the propensity to withdraw deposits from the bank, R_{D} is the rate of interest paid for deposits, D the volume of deposits at R_{D}, C_{L} a loan administration cost and C_{D} a deposit handling cost, then the impact of deposit disintermediation upon total cost is given quite simply as

$$\mathrm{TC} = LC_{\mathrm{L}} + DR_{\mathrm{D}} + DC_{\mathrm{D}} + g(\cdot), \tag{2}$$

where

$$\begin{aligned} g &= 0 && \text{if } k \leqslant x \leqslant R_{\mathrm{D}}, \\ g &= Dw(x - R_{\mathrm{D}}) && \text{if } R_{\mathrm{D}} < x \leqslant K. \end{aligned}$$

If interest rates fall below R_{D} no withdrawal takes place and total cost is given by $LC_{\mathrm{L}} + DR_{\mathrm{D}} + DC_{\mathrm{D}}$. On the other hand, if x rises above R_{D}, then deposits are withdrawn in the proportion w, and the bank's total costs fall by $R_{\mathrm{D}}Dw$ in recognition of the loss of deposits. Simultaneously costs increase by xDw since the lost deposits must be replaced with funds borrowed at x so that with $x > R_{\mathrm{D}}$ we have

$$\mathrm{TC} = LC_{\mathrm{L}} + DR_{\mathrm{D}} + DC_{\mathrm{D}} + Dw(x - R_{\mathrm{D}}).$$

If expressions (1) and (2) are rewritten in terms of their joint partial expectations, we have

$$
\begin{aligned}
&R_{\mathrm{L}}L\int_{k}^{R_{\mathrm{L}}} f(x)\,\mathrm{d}x - LR_{\mathrm{L}}\int_{k}^{R_{\mathrm{L}}}\left[\int_{0}^{1} yf(y,x)\,\mathrm{d}y\right]\mathrm{d}x \\
&\quad + L\int_{k}^{R_{\mathrm{L}}} x\left[\int_{0}^{1} yf(y,x)\,\mathrm{d}y\right]\mathrm{d}x + R_{\mathrm{L}}L\int_{R_{\mathrm{L}}}^{K} f(x)\,\mathrm{d}x - LC_{\mathrm{L}}\int_{R_{\mathrm{L}}}^{K} f(x)\,\mathrm{d}x \\
&\quad - LC_{\mathrm{L}}\int_{k}^{R_{\mathrm{L}}} f(x)\,\mathrm{d}x - R_{\mathrm{D}}D\int_{k}^{R_{\mathrm{D}}} f(x)\,\mathrm{d}x - R_{\mathrm{D}}D\int_{R_{\mathrm{D}}}^{K} f(x)\,\mathrm{d}x \\
&\quad + R_{\mathrm{D}}D\int_{R_{\mathrm{D}}}^{K}\left[\int_{0}^{1} wf(w,x)\,\mathrm{d}w\right]\mathrm{d}x - D\int_{R_{\mathrm{D}}}^{K} x\left[\int_{0}^{1} wf(w,x)\,\mathrm{d}w\right]\mathrm{d}x \\
&\quad + R_{\mathrm{L}}L\int_{R_{\mathrm{L}}}^{K} f(x)\,\mathrm{d}x - LC_{\mathrm{L}}\int_{R_{\mathrm{L}}}^{K} f(x)\,\mathrm{d}x - LC_{\mathrm{L}}\int_{k}^{R_{\mathrm{L}}} f(x)\,\mathrm{d}x \\
&\quad - R_{\mathrm{D}}D\int_{k}^{R_{\mathrm{D}}} f(x)\,\mathrm{d}x - (L-E-D)\int_{k}^{K} xf(x)\,\mathrm{d}x. \qquad (3)
\end{aligned}
$$

We have included in expression (3) a balance sheet constraint that maintains if lending is greater than funds available $(L-E-D>0)$, then bank borrowing must take place at an "a priori" rate of $\int_k^K xf(x)\,\mathrm{d}x$.

The random variables x and y are assumed to have a continuous joint probability density function entirely defined on $[k,K]$ and $[0,1]$ respectively. [5] We maintain that y is inversely related to x. As interest rates fall, more and more borrowers become inclined to prepay their loans. At higher rates of interest borrowers become less inclined to prepay. One simple way of formalizing this idea is to assume that the relationship between x and y is defined as a bivariate normal where the conditional expectation of y is a linear function of x with a negative slope "b" and a positive intercept "a"; that is, $\mu_{y|x} = a + bx$. The values of a and b are regression coefficients and can be expressed as functions of the unconditional means (μ_x, μ_y), the standard deviations (σ_x, σ_y), and the correlation coefficient ρ:

$$
\begin{aligned}
a &= \mu_y - b\mu_x, \\
b &= \frac{\rho\sigma_y}{\sigma_x}.
\end{aligned}
$$

It must be emphasized that y is the *propensity* to prepay and $\mu_{y|x}$ is the conditional mean *propensity* to prepay. The prepayments themselves depend on R_{L}. If x is at an historical low and the propensity to prepay is very high $(a+bx, b<0)$, there will be no prepayments if $x > R_{\mathrm{L}}$. Even if R_{L} is very high there will be no prepayments if $x > R_{\mathrm{L}}$. Hence the range of x over which the propensity to prepay is translated into

[5] The random variable x has unbounded support in this paper. Consequently, problems of negative interest rates can arise. Throughout the paper, the non-negative constraint is ignored for simplicity. The probability of negative interest rates can be made arbitrarily small by the appropriate choice of the underlying statistical parameters that characterize the density function of x. The same arguments are invoked in regard to the random variables y and w which, respectively, represent the propensity to prepay loans and to withdraw deposits.

prepayments is given by $k \leqslant x \leqslant R_L$ and $\int_k^{R_L^*} \mu_{y|x} f(x)\,dx$ is the expected proportion of prepayments (EPP).

In order to recognize the statistical dependence of the propensity to prepay loans upon the rate of interest x, the joint probabilities $f(x,y)$ which appear in Eq. (3) were rewritten as the product of $f(y|x)f(x)$ and then this expression was integrated over y. The optimal lending rate is then obtained by taking the derivative of Eq. (3) with respect to R_L and setting the result to zero which yields [6]

$$L\left[1 - \int_k^{R_L^*} \mu_{y|x} f(x)\,dx\right] + \frac{\partial L}{\partial R_L^*}\left[R_L^* - C_L - \mu_x - \int_k^{R_L^*} (R_L^* - x)\mu_{y|x} f(x)\,dx\right] = 0. \tag{4a}$$

Rearranging we have

$$\left(L + \frac{\partial L}{\partial R_L^*} R_L^*\right)\left[1 - \int_k^{R_L^*} \mu_{y|x} f(x)\,dx\right] = \frac{\partial L}{\partial R_L^*}\left[C_L + \mu_x - \int_k^{R_L^*} x\mu_{y|x} f(x)\,dx\right]. \tag{4b}$$

Acknowledging the prepayment risk confounds the optimality condition for R_L^*, but in predictable ways. Consider the optimality condition for R_L^* if borrowers did not have the option to prepay the loan.

$$L + \frac{\partial L}{\partial R_L^*} R_L^* = \frac{\partial L}{\partial R_L^*}(C_L + \mu_x). \tag{5}$$

The bank simply equates the deterministically given marginal revenue to the marginal cost. Clearly, (4b) is a stochastic analog of (5) with the distinction being two integrals that represent partial expectation operators. [7] The first integral on the LHS of (4) is the expected proportion of loan prepayments. The term $[1 - \int_k^{R_L^*} \mu_{y|x} f(x)\,dx]$ recognizes that the optimality condition does not equate the marginal revenue with the marginal cost of the last dollar lent, as in (5), but rather R_L^* equates the MR to the MC of the last *expected* dollar lent. At the margin, $L + \frac{\partial L}{\partial R_L^*} R_L^*$ will be lost on expected prepays but the RHS integral in (4b), $\int_k^{R_L^*} x\mu_{y|x} f(x)\,dx$, accounts for the expected return to the reinvested prepaid loans. [8]

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[6] The second order conditions for the optimal loan rate can be obtained from the authors.

[7] Although this paper presumes that the profit maximizing bank is a monopolist, the optimality condition for R_L^* can be rewritten in terms of an endogenous market structure. These notes will be made available to the reader upon request.

[8] Maximizing the objective function in Eq. (3) yields Eq. (4a) optimality condition for R_L^*. The integral differential equation refuses to yield an explicit solution for the optimal lending rate. Examining (4a), it is clear that R_L^* arithmetically influences the first order condition in a number of ways. The decision variable appears in the limit of the integral, in the integrand, and in the determination of the slope of the loan demand schedule. Given the disparate appearances of the optimal loan rate in (4a) normalizing on R_L^* is impossible. Not being able to isolate the optimal rate setting on the left-hand side of the FOC condition mutes any immediate intuition we might otherwise gain. In addition, we must appeal to the implicit function theorem to perform our comparative static analysis. It is clear that simulating the bank's optimal output price would provide the reader with a good deal of insight.

3. Comparative static behavior of the optimal loan rate under prepayment risk

Given the optimizing conditions above, we now address the impact of parameter changes upon R_{L}^* under prepayment risk. We employ the implicit function theorem to characterize the comparative static behavior of the optimal lending rate. Thus we have for any parameter q,

$$\frac{\mathrm{d}R_{\mathrm{L}}^*}{\mathrm{d}q} = -\frac{\partial\,\mathrm{OC}(\cdot)/\partial q}{\partial\,\mathrm{OC}(\cdot)/\partial R_{\mathrm{L}}^*},$$

where the optimality condition $\mathrm{OC}(\cdot)$ is

$$\partial E(\Pi)/\partial R_{\mathrm{L}} = \mathrm{OC}(R_{\mathrm{L}}, L, \sigma_y, \sigma_x, \rho_{x,y}, \mu_x, \mu_y, \ldots) = 0.$$

From the second order condition, the derivative of the optimality condition with respect to R_{L}^*, or $\partial\,\mathrm{OC}(\cdot)/\partial R_{\mathrm{L}}^*$ is negative. Consequently, the sign of the derivative of R_{L}^* with respect to any parameter q takes the sign of the derivative of $\partial\,\mathrm{OC}(\cdot)/\partial q$. The partial derivative of the optimality condition with respect to the parameters at hand can be decomposed into an expected proportion prepaid effect and an expected cost of prepayment (ECP) effect as given by

$$\frac{\partial\,\mathrm{OC}(\cdot)}{\partial q} = -\left[L\frac{\partial\,\mathrm{EPP}}{\partial q} + \frac{\partial L}{\partial R_{\mathrm{L}}}\frac{\partial\,\mathrm{ECP}}{\partial q}\right],$$

where $\mathrm{EPP} = \int_k^{R_{\mathrm{L}}^*} \mu_{y|x} f(x)\,\mathrm{d}x$ and $\mathrm{ECP} = \int_k^{R_{\mathrm{L}}^*} (R_{\mathrm{L}}^* - x)\mu_{y|x} f(x)\,\mathrm{d}x$. [9]

The resolution of $\partial\,\mathrm{OC}(\cdot)/\partial q$ into $\partial\,\mathrm{EPP}/\partial q$ and $\partial\,\mathrm{ECP}/\partial q$ repeatedly provides insights into the comparative static behavior of R_{L}^*. For instance, if the expected proportion of prepayment $\int_k^{R_{\mathrm{L}}^*} \mu_{y|x} f(x)\,\mathrm{d}x$ increases in the face of an increase of q, then the optimizing bank would be forced to reduce the risk of prepayment $\int_k^{R_{\mathrm{L}}^*} f(x)\,\mathrm{d}x$ by reducing the range of x over which prepayments are possible by reducing the optimal loan rate. The reduction in loan rate clearly brings EPP back into line and accounts for $\mathrm{d}R_{\mathrm{L}}^*/\mathrm{d}q < 0$. If parameter q increases the expected cost of prepayment $\int_k^{R_{\mathrm{L}}^*} (R_{\mathrm{L}}^* - x)\mu_{y|x} f(x)\,\mathrm{d}x$, then by reducing R_{L}^* the optimizing bank will reduce not only the range of x over which prepayments are possible, but the bank also will reduce the integrand. In this case, the bank manages the risk of prepayment and the cost of prepayment $(R_{\mathrm{L}}^* - x)$.

3.1. The impact of unconditional propensity to prepay upon the optimal loan rate

An increase in the unconditional propensity of borrowers to prepay loans, μ_y, increases the expected proportion of prepayment as documented by [10]

[9] The comparative static behavior of R_{L}^* and R_{D}^* is examined with respect to 10 parameters. Eight of these derivatives can be rewritten in terms of a EPP and a ECP effect.

[10] In order to facilitate the comparative static analysis our decision variables, the probability density function of x was rewritten in terms of the standard normal variate z.

$$\frac{\partial\,\mathrm{EPP}}{\partial\mu_y} = F_z\left(\frac{R_{\mathrm{L}}^{*} - \mu_x}{\sigma_x}\right) > 0. \tag{6a}$$

At the same time, the infinitesimal increase in μ_y increases the expected cost of prepayment: [11]

$$\frac{\partial\,\mathrm{ECP}}{\partial\mu_y} = (R_{\mathrm{L}}^{*} - \mu_x)F_z\left(\frac{R_{\mathrm{L}}^{*} - \mu_x}{\sigma_x}\right) + \sigma_x f_z\left(\frac{R_{\mathrm{L}}^{*} - \mu_x}{\sigma_x}\right) > 0. \tag{6b}$$

Increases in both EPP and ECP force the optimizing bank to reduce R_{L}^{*} in order to decrease the risk and the cost of prepayment. Mathematical support for this conclusion comes from being able to write $\partial\,\mathrm{OC}(\cdot)/\partial\mu_y$ as [12]

$$-F_z\left(\frac{R_{\mathrm{L}}^{*} - \mu_x}{\sigma_x}\right)\left[1 + \frac{\partial L}{\partial R_{\mathrm{L}}^{*}}\frac{R_{\mathrm{L}}^{*}}{L}\right] + \frac{\partial L}{\partial R_{\mathrm{L}}^{*}}\frac{1}{L}\int_{(k-\mu_x)/\sigma_x}^{(R_{\mathrm{L}}^{*}-\mu_x)/\sigma_x}(\mu_x + z\sigma_x)f(z)\,\mathrm{d}z < 0. \tag{6c}$$

With $\partial\,\mathrm{OC}(\cdot)/\partial\mu_y$ being less than zero it follows that $\partial R_{\mathrm{L}}^{*}/\partial\mu_y < 0$.

3.2. The impact of level of interest rates upon the optimal loan rate

An incremental increase in the expected level of interest rates reduces the expected proportion of borrower prepayments as given by

$$\frac{\partial\,\mathrm{ECP}}{\partial\mu_x} = (-1/\sigma_x)(\mu_y + b(R_{\mathrm{L}}^{*} - \mu_x))f_z((R_{\mathrm{L}}^{*} - \mu_x)/\sigma_x) < 0. \tag{7a}$$

The expected cost of prepays also falls with an infinitesimal increase in μ_x:

$$\frac{\partial\,\mathrm{ECP}}{\partial\mu_x} = -\int_{(k-\mu_x)/\sigma_x}^{(R_{\mathrm{L}}^{*}-\mu_x)/\sigma_x}(\mu_y + bz\sigma_x)f(z)\,\mathrm{d}z < 0. \tag{7b}$$

These two effects allow the optimizing bank to tolerate more risk, to increase exposure to prepayment, and thus to increase R_{L}^{*}. This assertion is supported by the result that

[11] A number of our comparative static results involve the evaluation of the partial expectations of the first and second moments of x. These details will be provided by the authors upon written request.

[12] The elasticity of the demand for loans is clearly negative and should have an absolute value that is less than one.

$$\frac{\partial \mathrm{OC}(\cdot)}{\partial \mu_x} = -L\left[\frac{-1}{\sigma_x}\left[\mu_y + b\left[\frac{R_L^* - \mu_x}{\sigma_x}\right]\sigma_x\right] f_z\left(\frac{R_L^* - \mu_x}{\sigma_x}\right)\right] + \frac{\partial L}{\partial R_L^*}\left[-1 - \int_{(k-\mu_x)/\sigma_x}^{(R_L^*-\mu_x)/\sigma_x} (-1)[\mu_y + bz\sigma_x] f(z)\,\mathrm{d}z\right] > 0, \tag{7c}$$

so that $\partial R_L^*/\partial \mu_x > 0$. [13]

3.3. The impact of interest rate volatility upon the optimal loan rate

The more casual reader might believe that an increase in σ_x will decrease R_L^* since the partial derivative of $\int_k^{R_L^*} xf(x)\,\mathrm{d}x$ with respect to σ_x is negative. Clearly an increase in σ_x reduces the expectation of x over the range of $f(x)$ that corresponds to prepayments. This effect *alone* would seem to bode a reduction in R_L^* with the increase in σ_x (recall above we found $\partial R_L^*/\partial \mu_x > 0$). However, consider $b = \frac{\rho\sigma_y}{\sigma_x}$ in the

[13] The relationship between the level of interest rates and prepayments is a crucial aspect of our modeling and is represented by $\mu_{y|x} = a + bx$. If the level of rates, x, is greater than R_L, then no loan prepayments will occur; if x is less than R_L, some prepayments will occur where more occur with lower values of x.

An important issue is how will the parameters "a" and "b" vary for different types of loans which banks make. Substantial differences in "a" due to differential costs of refinancing (including search and information costs) and loan maturity can make a large difference in the optimal loan rate the bank should charge. In general, greater costs of refinancing result in a lower value of "a" such that prepayments decrease. Thus when banks charge a greater fee for processing a loan (re)application, refinancings are lower and less likely.

The parameter "a" also varies with loan maturity. That is, if a loan has a short maturity, then the advantage to replacing debt with lower rate debt is less than for longer term loans as the lower rate renders a cost advantage for a shorter time. Many bank commercial loans are for less than 90 days while others are called "term" loans with maturities potentially greater than a year. Of course, many consumer loans may have maturities of less than 90 days while consumer automobile loans may have a maturity of several years.

Analysis of variation in "b" can be conducted by examining its component parts where $b = \rho\sigma_x\sigma_y/\sigma_x^2$; the slope is negative because ρ is negative. The greater the volatility of propensity to prepay, the more negative the slope. The correlation offers an opportunity for interesting analysis. If $x < R_L$, then one would expect a large proportion of the loans to prepay but some types of loans may have relatively weak correlation. Consider loans requiring collateral where the value of collateral is volatile. If interest rates decline and the value of collateral also dramatically declines, then the bank may not be willing to create a new loan even though the firm may request a refinancing. Or, the bank may be more demanding in terms of covenants for a new loan so that it is undesirable to the firm to replace the original loans. The likelihood of such a sequence of events is enhanced in that declining interest rates often occur at the same time the economy weakens; that is, the decline in interest rates may accompany a weakening economy as inflation declines and demand for loan declines. Also, consider loans that were originally made to firms with weak credit quality. If interest rates decline and firm credit quality simultaneously declines, the firm may not be able to get a new loan from banks given its credit quality may have declined.

In contrast to the above types of situations, noncollateralized loans to strong firms with superior credit quality would likely be able to refinance at will, even in the face of a weak economy, and thus analysts would find that prepayments and the level of interest rates have a stronger negative correlation. Bester (1994) develops a model where collateral requirements make it more likely the initial debt contract will be renegotiated in times of financial distress.

characterization of $\mu_{y|x} = a + bx$. Clearly an increase in σ_x diminishes the impact of the existing negative relationship between x and y. Surprisingly, this is advantageous to the bank and increases R_L^*. The bank wants x and y to be positively related (given the range of $\mu_{y|x}$ is between 0 and 1) and wants increases (decreases) in x to be associated with increases (decreases) in y. In this case, when the rate of interest is high, the propensity to prepay is high, and when the rate of interest is low, the propensity to prepay is low. This is a very desirable scenario for the bank because if x is high, relative to R_L^*, the high propensity to prepay is not realized. When x is low relative to R_L^*, only a small proportion of y is translated into prepayments by borrowers. Clearly the bank would like the correlation coefficient to be positive but rational behavior dictates a negative relation. However, it is true that the increase in σ_x dampens the adverse effect of ρ being negative and encourages greater R_L^*. The derivative $\partial R_L^*/\partial\sigma_x$ is positive because the $\partial\,\mathrm{OC}(\cdot)/\partial\sigma_x$ can be written as

$$L\left[\frac{R_L^* - \mu_x}{\sigma_x^2}\right][\mu_y + b(R_L^* - \mu_x)]f_z\left(\frac{R_L^* - \mu_x}{\sigma_x}\right) + \frac{\partial L}{\partial R_L^*}\mu_y(-1)f_z\left(\frac{R_L^* - \mu_x}{\sigma_x}\right)$$
$$+ \frac{\partial L}{\partial R_L^*}\rho\sigma_y \int_{(k-\mu_x)/\sigma_x}^{(R_L^*-\mu_x)/\sigma_x} z^2 f(z)\,\mathrm{d}z, \tag{8}$$

which is clearly a positive number.

Of course, the resolution of the sign of $\mathrm{d}R_L^*/\mathrm{d}\sigma_x$ does not speak to the magnitude of the impact of σ_x upon the optimal lending rate. In order to determine the sensitivity of R_L^* to changes in the volatility of the benchmark rate of interest, numeric solutions for the optimal lending rate were obtained under alternative specifications of σ_x. The results are provided in Fig. 1. R_L^* is 10% at $\sigma_x = 3.5\%$. The optimal lending rate falls as σ_x falls and increases as σ_x increases confirming the analytics. R_L^* is nearly linear in σ_x and is positive throughout the range of σ_x. [14]

3.4. The impact of prepayment volatility upon the optimal loan rate

Examining the optimality condition for R_L^* in Eq. (4) reveals the parameter σ_y is embedded in $\mu_{y|x} = a + bx$ where $b = \rho\sigma_y/\sigma_x$. The effect of an infinitesimal increase in σ_y is to augment the impact of ρ upon R_L^*. Recall ρ characterizes the inverse relationship between interest rates x and the propensity to prepay y. Magnifying the effect of ρ is to the detriment of the bank since the bank is better suited to handle a positive relationship between x and y, as discussed earlier. As expected, an increase in σ_y increases the expected proportion of prepayment as given by

$$\frac{\partial\,\mathrm{EPP}}{\partial\sigma_y} = -\rho f_z\left(\frac{R_L^* - \mu_x}{\sigma_x}\right) > 0. \tag{9a}$$

[14] A document providing the assumptions and the algebraic details of obtaining numeric solutions for R_L^* is available upon request.

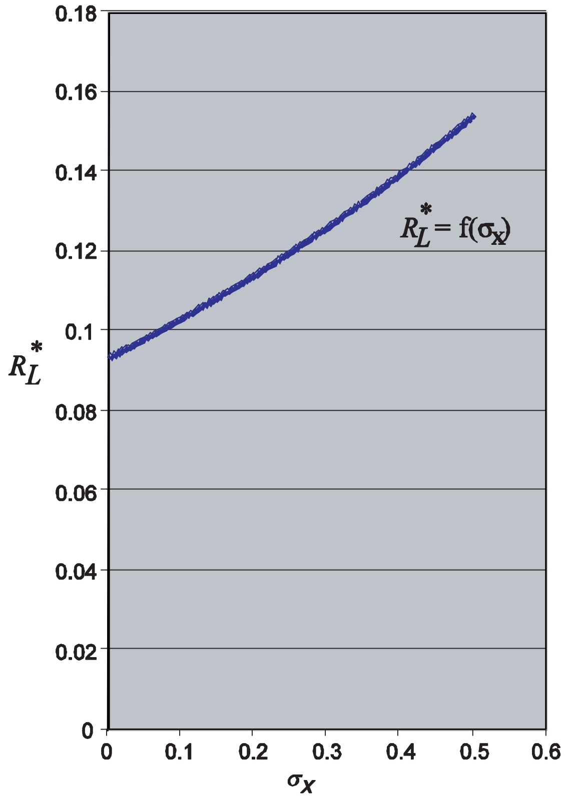

Fig. 1. Sensitivity of optimal loan rate to volatility of interest rates.

The expected cost of prepayment also increases with σ_y:

$$\frac{\partial\,\mathrm{ECP}}{\partial\sigma_y} = \rho\left[-(R_\mathrm{L}^* - \mu_x)f_z\left(\frac{R_\mathrm{L}^* - \mu_x}{\sigma_x}\right) - \sigma_x\int_{(k-\mu_x)/\sigma_x}^{(R_\mathrm{L}^*-\mu_x)/\sigma_x} z^2 f(z)\,\mathrm{d}z\right] > 0. \tag{9b}$$

Increases in both EPP and ECP suggests that $\partial R_\mathrm{L}^*/\partial\sigma_y$ will be negative. However, the sign of $\partial R_\mathrm{L}^*/\partial\sigma_y$ depends upon $\partial\,\mathrm{OC}(\cdot)/\partial\sigma_y$ given below:

$$\frac{\partial\,\mathrm{OC}(\cdot)}{\partial\sigma_y} = \frac{1}{R_\mathrm{L}^*}\rho\sigma_x L F_z\left(\frac{R_\mathrm{L}^* - \mu_x}{\sigma_x}\right)\left[\frac{\partial F_z\left(\frac{R_\mathrm{L}^* - \mu_x}{\sigma_x}\right)}{\partial R_\mathrm{L}^*}\frac{R_\mathrm{L}^*}{F_z\left(\frac{R_\mathrm{L}^* - \mu_x}{\sigma_x}\right)} + \frac{\partial L}{\partial R_\mathrm{L}^*}\frac{R_\mathrm{L}^*}{L}\right]. \tag{9c}$$

The sign of (9c) clearly depends upon the relative size of two elasticities: a risk elasticity and a loan demand elasticity. The elasticity $\partial F_z(\frac{R_\mathrm{L}^*-\mu_x}{\sigma_x})/\partial R_\mathrm{L}^* \cdot [R_\mathrm{L}^*/F_z(\frac{R_\mathrm{L}^*-\mu_x}{\sigma_x})]$ is the percentage change in prepayment risk relative to the percentage change in R_{L}^*. Clearly

$$F_z\left(\frac{R_{\mathrm{L}}^* - \mu_x}{\sigma_x}\right) = \int_{(k-\mu_x)/\sigma_x}^{(R_{\mathrm{L}}^*-\mu_x)/\sigma_x} f(z)\,\mathrm{d}z$$

is the likelihood that the rate of interest x will be less than R_{L}^*, i.e. the likelihood that any prepayment will take place. The derivative of $F_z(\frac{R_{\mathrm{L}}^*-\mu_x}{\sigma_x})$ with respect to R_{L}^* documents the increase in the risk of prepayment associated with increases in the optimal lending rate. With $\partial F_z(\frac{R_{\mathrm{L}}^*-\mu_x}{\sigma_x})/\partial R_{\mathrm{L}}^* = \frac{1}{\sigma_x} f_z(\frac{R_{\mathrm{L}}^*-\mu_x}{\sigma_x})$, prepayment risk is monotonically increasing in R_{L}^*. Unfortunately, the elasticity of prepayment risk cannot be so easily characterized. We have documented the behavior of $\frac{\partial F_z(\cdot)}{\partial R_{\mathrm{L}}^*} \cdot \frac{R_{\mathrm{L}}^*}{F_z(\cdot)}$ for four sets of $[\mu_x, \sigma_x]$ and thirteen different R_{L}^*. All the parameters are arbitrarily chosen. However, the selections for μ_x and σ_x are thought to be realistic while the optimal lending rates are across a wider spectrum.

In order to get a sense of the behavior of the prepayment risk elasticity, R_{L}^* was given uncharacteristically small and large as well as reasonable illustrative values. For all four panels, at low lending rates the risk elasticities began low but rose to their maximums as R_{L}^* neared the mean value of x. At values of R_{L}^* unrealistically greater than μ_x, the risk elasticities recede to small positive numbers. Though the risk elasticities are highly variable if the selected R_{L}^* is realistic, say within one standard deviation of μ_x, $\frac{\partial F_z(\cdot)}{\partial R_{\mathrm{L}}^*} \cdot \frac{R_{\mathrm{L}}^*}{F_z(\cdot)}$ is a large positive number generally close to one. [15] At the same time, the elasticity of loan demand $\frac{\partial L}{\partial R_{\mathrm{L}}^*} \frac{R_{\mathrm{L}}^*}{L}$ is negative and the absolute value of its magnitude is necessarily less than one.

Under these conditions, the linear combination of the two elasticities yields a positive number making $\partial\,\mathrm{OC}(\cdot)/\partial\sigma_y$ and $\partial R_{\mathrm{L}}^*/\partial\sigma_y$ negative. Since both $\partial\,\mathrm{EPP}/\partial\sigma_y$ and $\partial\,\mathrm{ECP}/\partial\sigma_y$ were positive, the reduction in R_{L}^* with an increase in σ_y is not surprising. But the role of the prepayment risk elasticity is a particularly satisfying aspect of the analysis. The optimal rate of lending is only reduced and earnings are only foregone if the associated decrease in prepayment risk is significant. For example, if $(R_{\mathrm{L}}^* - \mu_x)/\sigma_x$ is very small or very large, then $\partial F_z(\frac{R_{\mathrm{L}}^*-\mu_x}{\sigma_x})/\partial R_{\mathrm{L}}^*$ will be quite small, the elasticity will be small and $\partial R_{\mathrm{L}}^*/\partial\sigma_y$ will not be negative. A marginal decrease in the lending rate will not reduce prepayment risk enough to justify the earnings that must be foregone at the new lower R_{L}^*.

3.5. The impact of the correlation between interest rates and prepayments upon optimal loan rate

As established earlier in our discussion of the comparative static behavior of R_{L}^*, the inverse relationship between interest rates and the propensity to prepay works to the disadvantage of banks. An optimizing bank would like x and y to be positively related, in which case, when x is large, y is large. But the large propensity to prepay

[15] Tables documenting the behavior of the elasticity of the risk of prepayment will be made available to the reader upon request.

may go unrealized if x is greater than R_L^*. From the bank's point of view, any increase in ρ (to something less negative) would be a good thing. The partial derivative of EPP with respect to ρ is negative as given by

$$\frac{\partial\,\mathrm{EPP}}{\partial\rho} = -\sigma_y f_z\left(\frac{R_L^* - \mu_x}{\sigma_x}\right) < 0. \tag{10a}$$

The partial derivative of ECP with respect to ρ is also negative:

$$\frac{\partial\,\mathrm{ECP}}{\partial\rho} = \left[-(R_L^* - \mu_x)f_z\left(\frac{R_L^* - \mu_x}{\sigma_x}\right) - \int_{(k-\mu_x)/\sigma_x}^{(R_L^*-\mu_x)/\sigma_x} z^2 f(z)\,\mathrm{d}z\right]\sigma_y < 0. \tag{10b}$$

Finally the derivative of the optimality condition with respect to ρ is given as

$$\frac{\partial\,\mathrm{OC}(\cdot)}{\partial\rho} = \frac{\sigma_x\sigma_y L F_z\left(\frac{R_L^* - \mu_x}{\sigma_x}\right)}{R_L^*}\left[\frac{\partial F_z\left(\frac{R_L^* - \mu_x}{\sigma_x}\right)}{\partial R_L^*}\cdot\frac{R_L^*}{F_z\left(\frac{R_L^* - \mu_x}{\sigma_x}\right)} + \frac{\partial L}{\partial R_L^*}\frac{R_L^*}{L}\right] > 0. \tag{10c}$$

$\partial\,\mathrm{OC}(\cdot)/\partial\rho$ is only positive if the elasticity of prepayment risk is once again greater than the absolute value of the elasticity of the demand for loans. This is true whenever R_L^* is near the mean of x; that is, whenever R_L^* is realistic. The $\partial\,\mathrm{OC}(\cdot)/\partial\rho$ is only positive if the increase in ρ transpires at a statistically meaningful time, when $\partial F_z(\cdot)/\partial R_L$ is large. This is when the increase in ρ contributes to the resolution of the prepayment risk, $F_z(\frac{R_L^*-\mu_x}{\sigma_x})$. With $\partial\,\mathrm{OC}(\cdot)/\partial\rho > 0$, then $\partial R_L^*/\partial\rho$ is also greater than zero.

4. Deposit withdrawals

Just as loans suffer from prepayment risk, deposits are subject to the risk of early withdrawal. Recall, w is the propensity of depositors to withdraw and is thought to be a positive function of x, the level of interest rates. As interest rates fall, depositors become less inclined to withdraw their deposits. At higher rates of interest, depositors become more inclined to withdraw. The variates are assumed to have a continuous joint probability density function defined on $[k, K]$ and $[0, 1]$ and the relationship between x and w is given by a bivariate normal where $\mu_{w|x} = c + \mathrm{d}x$ with both c and d being greater than zero. It must be emphasized that w is the *propensity* to withdraw deposits and $\mu_{w|x}$ is the conditional mean *propensity* to withdraw deposits. The withdrawals themselves ultimately depend upon R_D. If x is at an all time high and the propensity to withdraw is very high (near 1) there will be no deposit withdrawals if $R_D > x$. Even if R_D is very low there will be no withdrawals if $R_D > x$. Hence the range of x over which the propensity to withdraw is translated into a realized withdrawal is given by $R_D^* \leqslant x \leqslant K$ and $\int_{R_D^*}^{K} f(x)\,\mathrm{d}x$ is the risk of early withdrawals. Maximizing (3) with respect to the optimal input price R_D^* yields

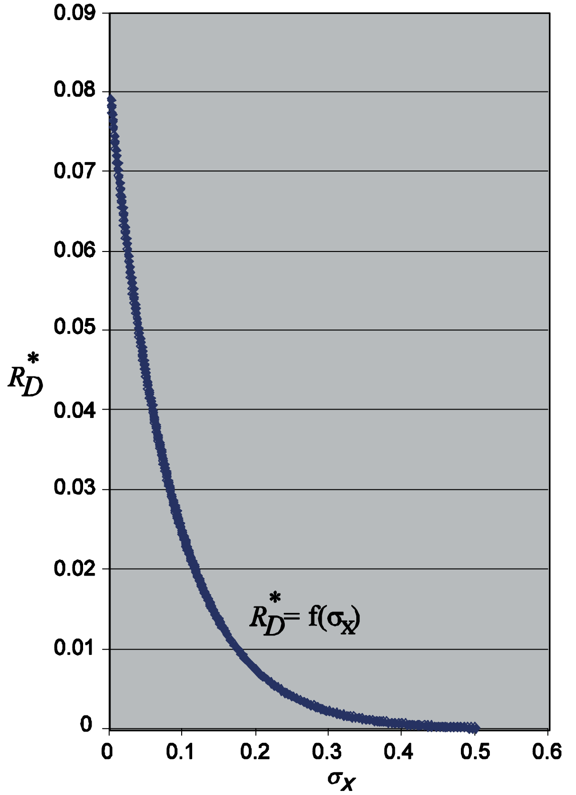

Fig. 2. Sensitivity of optimal deposit rate to volatility of interest rates.

$$\mathrm{OC}(\cdot) = -D\left[1 - \int_{R_{\mathrm{D}}^*}^{K} \mu_{w|x} f(x)\,\mathrm{d}x\right] - \frac{\partial D}{\partial R_{\mathrm{D}}^*}\left[R_{\mathrm{D}}^* - C_{\mathrm{D}} - \mu_x - \int_{R_{\mathrm{D}}^*}^{K} (R_{\mathrm{D}}^* - x)(\mu_{w|x}) f(x)\,\mathrm{d}x\right] = 0. \tag{11}$$

The optimality condition for R_{D}^* in (11) is the mirror reflection of the optimality condition for R_{L}^* with R_{D}^* replacing R_{L}^*, $\mu_{w|x}$ replacing $\mu_{y|x}$, C_{D} replacing C_{L} and $\partial D/\partial R_{\mathrm{D}}^*$ replacing $\partial L/\partial R_{\mathrm{L}}^*$. [16] Given the similarity in the scenarios surrounding the

[16] Maximizing the objective function in Eq. (3) yields Eq. (11), the optimality condition for R_{D}^*. The integral differential equation above refuses to yield an explicit solution for the optimal deposit rate. Examining (11), it is clear that R_{D}^* arithmetically influences the first order condition in a number of ways. The decision variable appears in the limit of integral, in the integrand, and in the determination of slope of the deposit supply schedule. Given the disparate appearances of the optimal deposit rate in (11) normalizing on R_{D}^* is impossible. Not being able to isolate the optimal rate setting on the left hand side of the FOC condition mutes any immediate intuition we might otherwise gain. In addition, we must appeal to the implicit function theorem to perform our comparative static analysis. It is clear that simulating the bank's optimal input price would provide the reader with a good deal of insight.

determination of R_L^* and R_D^*, the symmetry in the optimality condition is not a surprise.

Although the mechanics of the comparative static behavior of R_D^* are perfectly analogous to those of R_L^*, the results will be considered here for the insights they provide to the management of the risk of withdrawal. An increase in the unconditional mean propensity to withdraw deposits μ_w increases R_D^*. The change in μ_w increases the expected proportion of withdrawals so that $\partial \mathrm{OC}(\cdot)/\partial \mu_w > 0$. The bank reacts to the increase in the unconditional mean of w by reducing the risk of withdrawal where $\int_{R_D^*}^K f(x)\,dx$, that is $\partial R_D^*/\partial \mu_w > 0$. An incremental increase in μ_x, the mean level of interest rates, increases the expected proportion of withdrawals $\int_{R_D^*}^K \mu_{w|x} f(x)\,dx$.

Consequently, the bank reduces the risk of withdrawal by increasing the price paid for deposits. In other words, $\partial R_D^*/\partial \mu_w > 0$.

In considering the partial derivative of R_D^* with respect to σ_x, it must be recalled that the regression coefficient $d(=\frac{\phi\sigma_w}{\sigma_x})$ in $\mu_{w|x} = c + dx$ is positive because $\phi > 0$. The positive relationship between x and w is to the disadvantage of the bank. When interest rates are high, the propensity to withdraw is high. That is, when banks have their greatest exposure to the possibility of withdrawals (high x) the propensity to withdraw is its highest. Under these circumstances, from the bank's point of view, an increase in σ_x is good because it serves to diminish the importance of ϕ in the determination of $d(=\frac{\phi\sigma_w}{\sigma_x})$. In fact, an increase in σ_x does decrease the proportion of depositors expected to withdraw. Thus, the optimizing bank tolerates more withdrawal risk and pays less for its inputs as $\partial R_D^*/\partial \sigma_x < 0$. The comparative static relationship between σ_x and R_D^* is graphically depicted in Fig. 2.

An infinitesimal increase in σ_w serves to worsen the adverse impact that ϕ has upon the bank because $d = \frac{\phi\sigma_w}{\sigma_x}$. An increase in σ_w increases the proportion of depositors expected to withdraw. Optimally, the bank reduces its disintermediation risk by increasing R_D^* but only if

$$\frac{\partial \mathrm{OC}(\cdot)}{\partial \sigma_w} = \frac{\sigma_x \phi D}{R_D^*}[1 - F_z(\cdot)][-1]\left[\frac{\partial[1 - F_z(\cdot)]}{\partial R_D^*}\frac{R_D^*}{[1 - F_z(\cdot)]} + \frac{\partial D}{\partial R_D^*}\frac{R_D^*}{D}\right] > 0. \tag{12}$$

The sign of (12) clearly depends upon the relative size of two elasticities: a risk elasticity and a deposit supply elasticity. The elasticity $\partial[1 - F_z(\frac{R_D^* - \mu_x}{\sigma_x})]/\partial R_D^* \cdot [R_D^*/[1 - F_z(\frac{R_D^* - \mu_x}{\sigma_x})]]$ is the percentage change in the risk of withdrawal relative to the percentage change in R_D^*. Clearly, $\partial[1 - F_z(\frac{R_D^* - \mu_x}{\sigma_x})]/\partial R_D^*$ is negative and, consequently, the risk elasticity is negative. Increases in R_D^* secure more deposits for the bank so that $\partial D/\partial R_D^*$ is positive and the deposit supply elasticity is a positive number. A linear combination of the two elasticities will be negative, if R_D^* is realistic, making $\partial \mathrm{OC}(\cdot)/\partial \mu_w$ and $\partial R_D^*/\partial \sigma_w$ positive. The increase in R_D^* with an increase in σ_w is not surprising given σ_w's impact upon the regression coefficient "d". But the role of the withdrawal risk elasticity is a reassuring aspect of the analysis. The optimal rate paid for deposits is only increased if the associated decrease in the risk of withdrawal is significant. For example, if R_D^* is unrealistically large or unrealistically small then $\partial[1 - F_z(\frac{R_D^* - \mu_x}{\sigma_x})]/\partial R_D^*$ will be a small negative number, the risk elasticity will be a small negative number, and $\partial R_D^*/\partial \sigma_w$ will not be positive. A marginal increase in the deposit rate will not reduce the risk of withdrawal enough to justify the increase in input costs.

Table 1
Summary of the comparative static behavior of optimal loan and deposit rates

$\frac{\partial R_{\mathrm{L}}^*}{\partial \mu_y} < 0$	$\frac{\partial R_{\mathrm{D}}^*}{\partial \mu_w} > 0$
$\frac{\partial R_{\mathrm{L}}^*}{\partial \mu_x} > 0$	$\frac{\partial R_{\mathrm{D}}^*}{\partial \mu_x} > 0$
$\frac{\partial R_{\mathrm{L}}^*}{\partial \sigma_x} > 0$	$\frac{\partial R_{\mathrm{D}}^*}{\partial \sigma_x} < 0$
$\frac{\partial R_{\mathrm{L}}^*}{\partial \sigma_y} < 0$	$\frac{\partial R_{\mathrm{D}}^*}{\partial \sigma_w} > 0$
$\frac{\partial \mathrm{OC}(\cdot)}{\partial \sigma_y} = \frac{\rho \sigma_x L}{R_{\mathrm{L}}^*} F_z\left(\frac{R_{\mathrm{L}}^* - \mu_x}{\sigma_x}\right) \cdot \left[\frac{\partial F_z\left(\frac{R_{\mathrm{L}}^* - \mu_x}{\sigma_x}\right)}{\partial R_{\mathrm{L}}^*} \frac{R_{\mathrm{L}}^*}{F_z\left(\frac{R_{\mathrm{L}}^* - \mu_x}{\sigma_x}\right)} + \frac{\partial L}{\partial R_{\mathrm{L}}^*} \frac{R_{\mathrm{L}}^*}{L}\right] < 0$	$\frac{\partial \mathrm{OC}(\cdot)}{\partial \sigma_w} = \frac{(-1)\sigma_x \phi}{R_{\mathrm{D}}^*} D\left[1 - F_z\left(\frac{R_{\mathrm{D}}^* - \mu_x}{\sigma_x}\right)\right] \cdot \left[\frac{\partial\left(1 - F_z\left(\frac{R_{\mathrm{D}}^* - \mu_x}{\sigma_x}\right)\right)}{\partial R_{\mathrm{D}}^*} \cdot \frac{R_{\mathrm{D}}^*}{\left(1 - F_z\left(\frac{R_{\mathrm{D}}^* - \mu_x}{\sigma_x}\right)\right)} + \frac{\partial D}{\partial R_{\mathrm{D}}^*} \cdot \frac{R_{\mathrm{D}}^*}{D}\right] > 0$
$\frac{\partial R_{\mathrm{L}}^*}{\partial \rho} > 0$	$\frac{\partial R_{\mathrm{D}}^*}{\partial \phi} > 0$
$\frac{\partial \mathrm{OC}(\cdot)}{\partial \rho} = \frac{\sigma_x \sigma_y L F_z\left(\frac{R_{\mathrm{L}}^* - \mu_x}{\sigma_x}\right)}{R_{\mathrm{L}}^*} \cdot \left[\frac{\partial F_z\left(\frac{R_{\mathrm{L}}^* - \mu_x}{\sigma_x}\right)}{\partial R_{\mathrm{L}}^*} \cdot \frac{R_{\mathrm{L}}^*}{F_z\left(\frac{R_{\mathrm{L}}^* - \mu_x}{\sigma_x}\right)} + \frac{\partial L}{\partial R_{\mathrm{L}}^*} \frac{R_{\mathrm{L}}^*}{L}\right] > 0$	$\frac{\partial \mathrm{OC}(\cdot)}{\partial \phi} = \frac{\sigma_x \sigma_w D}{R_{\mathrm{D}}^*}\left[1 - F_z\left(\frac{R_{\mathrm{D}}^* - \mu_x}{\sigma_x}\right)\right][-1] \cdot \left[\frac{\partial\left[1 - F_z\left(\frac{R_{\mathrm{D}}^* - \mu_x}{\sigma_x}\right)\right]}{\partial R_{\mathrm{D}}^*} \frac{R_{\mathrm{D}}^*}{\left[1 - F_z\left(\frac{R_{\mathrm{D}}^* - \mu_x}{\sigma_x}\right)\right]} + \frac{\partial D}{\partial R_{\mathrm{D}}^*} \frac{R_{\mathrm{D}}^*}{D}\right] > 0$

As established earlier, the positive relationship between interest rates and the propensity to withdraw deposits works to the disadvantage of banks. An optimizing bank would like x and w to be inversely related, in which case, when x is small then w is large. But the large propensity to withdraw may go unrealized if x is so low that it is less than R_{D}. From the bank's point of view any increase in ϕ is a undesirable. As expected, an increase in ϕ increases the proportion of depositors expected to withdraw. The optimizing bank reduces its risk of withdrawal by increasing R^*_{D} if

$$\frac{\partial \mathrm{OC}(\cdot)}{\partial \phi} = \frac{\sigma_x \sigma_w D}{R^*_{\mathrm{D}}}[1 - F_z(\cdot)][-1]\left[\frac{\partial[1 - F_z(\cdot)]}{\partial R^*_{\mathrm{D}}}\frac{R^*_{\mathrm{D}}}{[1 - F_z(\cdot)]} + \frac{\partial D}{\partial R^*_{\mathrm{D}}}\frac{R^*_{\mathrm{D}}}{D}\right] > 0. \tag{13}$$

$\partial \mathrm{OC}(\cdot)/\partial \phi$ is only positive if the linear combination of the risk and deposit supply elasticities is a negative number. This is true whenever R^*_{D} is near the mean of x; that is, whenever R^*_{D} is realistic. The derivative $\partial R^*_{\mathrm{D}}/\partial \phi$ is positive if the increase in ϕ transpires at a statistically meaningful time which is when the absolute value of $\partial[1 - F_z(\frac{R^*_{\mathrm{D}} - \mu_x}{\sigma_x})]/\partial R^*_{\mathrm{D}}$ is large. See Table 1 for a summary of these comparative static results.[17]

5. Conclusion

Bank customers enjoy options to prepay loans and to withdraw deposits. This paper has detailed the impact of these embedded options upon the optimal intermediation margin. The objective function was formulated so that the revenues from making a loan depended upon a rational process for borrower prepayments. The bank's cost of acquiring funds depended upon a process for depositor withdrawal. Although the optimality conditions for loan and deposit rates were relatively abstruse, the comparative static behavior of the optimal output and input prices was intuitive. An increase in the mean propensity to either prepay loans or withdraw deposits forced the bank to reduce the optimal lending rate and pay more for deposits. An increase in the mean level of interest rates increased both optimal loan and deposit rates. The reduction in the likelihood of prepayment encouraged the bank to increase the optimal lending rate and the increase in the risk of withdrawal forced the bank to pay more for deposits. The net impact upon the intermediation margin is ambiguous since the change in the spread depends upon the relative magnitudes of a

[17] Deterministic relations between the benchmark rate "x" and the respective dependent variables "y" and "w" can be portrayed as special cases. For example, if the reader is convinced that "y" is nonstochastically given by "x" then, when $x < R_{\mathrm{L}}$, prepayments take place in the magnitude of $y = \mu_{y|x} = a + bx$ where $b = \frac{\rho\sigma_y}{\sigma_x} = \frac{(-1)\sigma_y}{\sigma_x}$. Clearly, a deterministic relation between benchmark rate "x" and the propensity to prepay is just a special case of the model where $\rho = -1$. Alternatively, when $0 > \rho > -1$ with $x < R_{\mathrm{L}}$, prepayments will place it a magnitude slightly more or slightly less than "$a + bx$" depending upon just how close ρ is to (−1). This case recognizes that other unacknowledged variables, including transaction costs, can influence loan prepayments. Clearly, our model also accommodates the possibility that the propensity to disintermediate "w" can be given by the benchmark rate "x" with certainty. In this case, ϕ would be given a value of 1.

host of parameters contained in $\partial\,\mathrm{OC}(R_\mathrm{L}^*,\ldots)/\partial\mu_x$, $\partial\,\mathrm{OC}(R_\mathrm{D}^*,\ldots)/\partial\mu_x$, $\partial\,\mathrm{OC}(R_\mathrm{L}^*,\ldots)/\partial R_\mathrm{L}^*$, and $\partial\,\mathrm{OC}(R_\mathrm{D}^*,\ldots)/\partial R_\mathrm{D}^*$.

An increase in interest rate volatility diminished the adverse impact of both the correlation between interest rates and prepayment and the correlation between interest rates and withdrawals. As a consequence, the optimal loan rate increased and the optimal deposit rate decreased, widening the optimal intermediation margin. An increase in volatility of prepayments served to worsen the adverse impact of the correlation between interest rates and propensity to prepay. Similarly, an increase in the volatility of withdrawals worsened the impact of the correlation between interest rates and the propensity to withdraw. The result was a decrease in optimal loan rate and an increase in optimal deposit rate. A positive correlation between interest rates and propensity to prepay would please an optimizing bank because any increase in such correlation increases the optimal loan rate. A negative correlation between interest rates and propensity to withdraw would be to the advantage of bank management because an increase in this correlation increases optimal deposit rates. [18]

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References

Ahn, D.H., Boudoukh, J., Richardson, M., Whitelaw, R.F., 1999. Optimal risk management using options. Journal of Finance 54, 359–375.

Allen, L., 1988. The determinants of bank interest margins: A note. The Journal of Financial and Quantitative Analysis 23, 231–235.

Angbazo, L., 1997. Commerical bank net interest margins, default risk, interest-rate risk, and off-balance sheet banking. Journal of Banking and Finance 21, 55–87.

[18] Theoretically, if interest rates were floating, the bank would not be subject to withdrawals and prepayments because there would not be motivation to prepay or withdraw. That is, if interest rates decreased (increased), the interest rate on loans (deposits) would be reset to some lower (higher) rate consistent with the new, lower (higher) level of interest rates. However, there could be some limited potential for prepays and withdrawals to the extent that, realistically, interest rates are not adjusted daily. Some theory of floating rate instruments maintains that the maturity of a floating rate instrument is the time until the interest rate is reset. If the regular time to reset is rather long, say six months, there may be potential to prepay a loan if the interest rate has declined a great deal. The shorter the time to reset, the less likely prepayments will occur. Also, there could be some limited potential for prepayment when the credit status of the borrower dramatically improves such that they can recontract at a lower interest rate reflecting lower default risk.

Baltensperger, E., 1980. Alternative approaches to the theory of the banking firm. Journal of Monetary Economics 6, 1–38.

Berger, A.N., Hannan, T.H., 1998. The efficiency cost of market power in the banking industry: A test of the 'quiet life' and related hypotheses. Review of Economics and Statistics 80, 454–465.

Berger, A.N., Bonime, S.D., Covitz, D.M., Hancock, D., 2000. Why are bank profits so persistent? The roles of product market competition, informational opacity, and regional/macroeconomic shocks. Journal of Banking and Finance 24, 1203–1235.

Bester, H., 1994. The role of collateral in a model of debt renegotiation. Journal of Money, Credit, and Banking 26, 72–86.

Hannan, T.H., 1991. Bank commercial loan markets and the role of market structure: Evidence from surveys of commercial lending. Journal of Banking and Finance 15, 133–149.

Ho, T., Saunders, A., 1981. The determinants of bank interest margins: Theory and evidence. Journal of Financial and Quantitative Analysis 16, 581–600.

Hutchison, D., Pennacchi, G., 1996. Measuring rents and interest rate risk in imperfect financial markets: The case of retail deposits. Journal of Financial and Quantitative Analysis 31, 399–417.

Madura, J., Zarruk, E.R., 1992. Optimal bank interest margin under capital regulation and deposit insurance. Journal of Financial and Quantitative Analysis 27, 143–149.

McShane, R.W., Sharpe, I.G., 1985. A time series/cross section analysis of the determinants of Australian trading bank loan/deposit interest margins: 1962–1981. Journal of Banking and Finance 9, 115–136.

Santomero, A.M., 1984. Modeling the banking firm: A survey. Journal of Money, Credit, and Banking 16, 576–602.

Wong, K.P., 1997. On the determinants of bank interest margins under credit and interest rate risks. Journal of Banking and Finance 21, 251–271.

Zarruk, E.R., 1989. Bank spread with uncertain deposit level and risk aversion. Journal of Banking and Finance 13, 797–810.